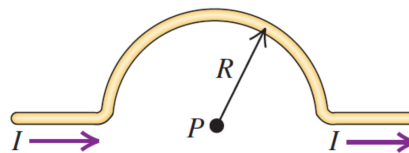


## Physics 1C Discussion (Week 2)

### Problem 1

**28.32 •** Calculate the magnitude and direction of the magnetic field at point  $P$  due to the current in the semicircular section of wire shown in **Fig. E28.32**. (*Hint: Does the current in the long, straight section of the wire produce any field at  $P$ ?*)

Figure E28.32



We use a cylindrical coordinate system  $(r, \theta, z)$  (See Appendix A). We cannot use Ampère's law because the problem has no axial symmetry. In such a problem with less symmetry, we have to employ the Biot-Savart Law, which is applicable to a more general system. Let's take the origin to be at  $P$  (so  $\hat{r}$  is a unit vector pointing away from the origin). Then the magnetic field  $\vec{B}$  at point  $P$  by Biot-Savart Law is

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times (-\hat{r})}{r^2}, \quad (1)$$

where  $d\vec{l}$  is an infinitesimal line element vector running along the current<sup>1</sup>, and  $r$  is the distance from point  $P$  to each tiny line element. We also set  $\hat{\theta}$  pointing along the anti-clockwise direction in this paper and  $\hat{z}$  pointing normal to this paper toward us.

Let's break this problem up into three parts.

- (1) The left part of the straight wire,
- (2) the center arc,
- (3) the right part of the straight wire

For parts 1) and 3),  $\hat{r}$  is (anti-)parallel to  $d\vec{l}$ , and  $\hat{r}$  is pointing away from the origin. As such, we know  $d\vec{l} \times \hat{r} = 0$ . So, there is no contribution from parts 1) and 3)

$$\vec{B}_1 = \vec{B}_3 = 0. \quad (2)$$

For part 2),  $d\vec{l}$  is going along the wire in the  $-\hat{\theta}$  direction<sup>2</sup>.

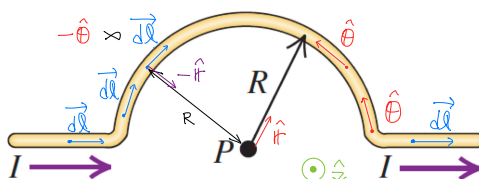


Figure 1: The setup of the unit vectors in the system

As such,

$$d\vec{l} \times (-\hat{r}) = dl(\hat{\theta} \times \hat{r}) = -dl\hat{z}. \quad (3)$$

As we are in cylindrical coordinates, we have  $dl = R d\theta$ , and  $r = R$  is constant. Then we yield

$$\vec{B}_2 = -\frac{\mu_0}{4\pi} \int_0^\pi \frac{I(R d\theta) \hat{z}}{R^2} = -\frac{\mu_0 I}{4R} \hat{z} \quad (4)$$

<sup>1</sup>See also <https://pressbooks.online.ucf.edu/osuniversityphysics2/chapter/the-biot-savart-law/>

<sup>2</sup>As a note, we are integrating from right to left, so  $d\vec{l}$  is going along that direction as well (see Fig 1).

If we had chosen to go from left to right, we would also have to flip the direction of  $d\vec{l}$

Finally, putting this together in the Biot-Savart Law, we get

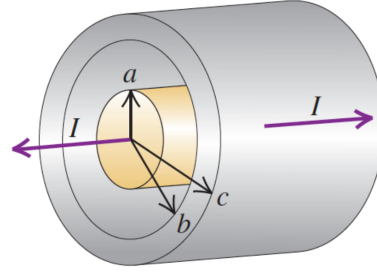
$$\begin{aligned}\vec{B} &= \vec{B}_1 + \vec{B}_2 + \vec{B}_3 \\ &= \vec{B}_2 \\ &= -\frac{\mu_0 I}{4R} \hat{z},\end{aligned}\tag{5}$$

where we have integrated  $\theta$  from 0 to  $\pi$ .

## Problem 2

**28.39 • Coaxial Cable.** A solid conductor with radius  $a$  is supported by insulating disks on the axis of a conducting tube with inner radius  $b$  and outer radius  $c$  (Fig. E28.39). The central conductor and tube carry equal currents  $I$  in opposite directions. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude of the magnetic field (a) at points outside the central, solid conductor but inside the tube ( $a < r < b$ ) and (b) at points outside the tube ( $r > c$ ).

Figure E28.39



We use a cylindrical coordinate system  $(r, \theta, z)$ , and set  $\hat{z}$  pointing along the inner current and  $\hat{\theta}$  pointing along the anticlockwise direction around  $\hat{z}$ . Since the problem has axial symmetry, we will use Ampère's law instead of Biot-Savart law.

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}. \quad (6)$$

For a), The surface we will use for Ampère's law is a cylinder in between  $r = a$  and  $r = b$ . Now for  $a < r < b$ ,  $I_{\text{enc}} = I$ . Also,  $d\vec{l} = r d\theta \hat{\theta}$ . And so,

$$\int_0^{2\pi} B r d\theta = 2\pi r B = \mu_0 I \quad (7)$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}. \quad (8)$$

For b), The surface we will use for Ampère's law is a cylinder outside of  $r = c$ . Then,  $I_{\text{enc}} = I + (-I) = 0$ , where we have added the inner cable's  $I$  to the outer cable's  $-I$  such that the total enclosed current is just zero. So, we have,

$$\int_0^{2\pi} B r d\theta = 2\pi r B = 0 \quad (9)$$

$$\Rightarrow B = 0. \quad (10)$$

This last result is very important! If we only had the inner cable, every wire would exert a magnetic field on the metal, electrons, and other wires nearby. This can cause serious problems when trying to design some electronics. However, including both wires (inner and outer shell), there is not magnetic interaction between wires!

### Background of this problem

The practicality of this coaxial cable is very high. The outer conductor covering the inner conductor (core wire) serves as an electromagnetic shield, making it less susceptible to electromagnetic waves arriving from the outside. It is mainly used as a cable for transmitting high-frequency signals in radio communication equipment, broadcasting equipment, network equipment, and electronic measuring instruments.

## A Cylindrical coordinate

Here, we outline the setup of the 3-dimensional cylindrical coordinate shown in Fig 2.

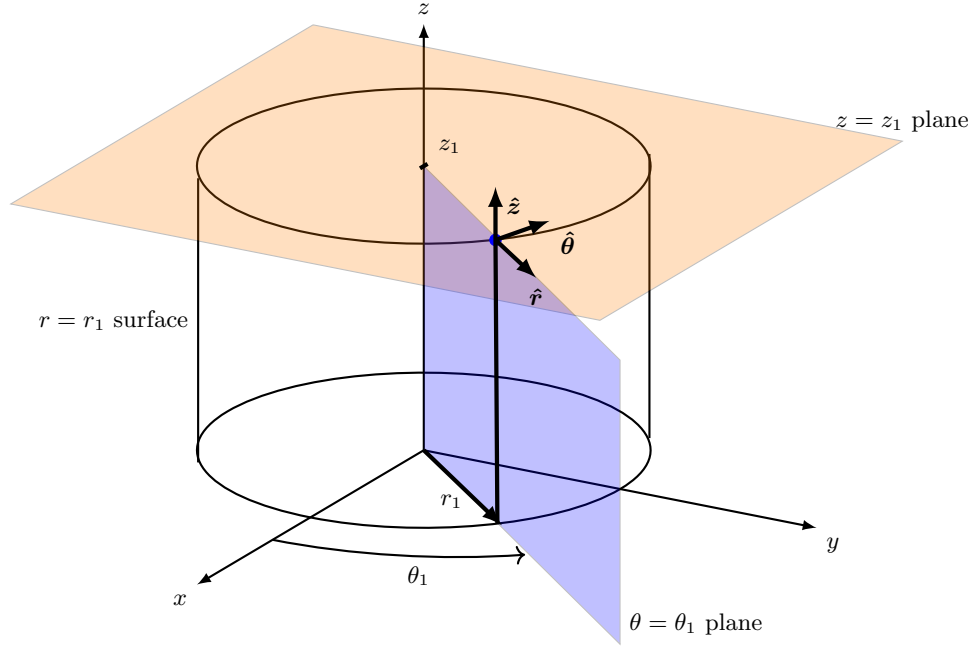


Figure 2: Comparison of the Cartesian coordinate system  $(x, y, z)$ , and the cylindrical coordinate system  $(r, \theta, z)$

The physical result does not depend on your choice of coordinate system. That means you can choose any coordinate for your problems. But, a certain type of “smart” choice exists for a particular situation. When setting a coordinate system, you must be careful about the *symmetry* associated with the problem.

When converting from 3-dimensional Cartesian coordinates to 3-dimensional cylindrical coordinate set as Fig 2, you need to perform the following transformation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \cos \theta \\ r \sin \theta \\ z \end{pmatrix}, \quad \begin{pmatrix} r \\ \theta \\ z \end{pmatrix} = \begin{pmatrix} \sqrt{x^2 + y^2} \\ \arctan\left(\frac{y}{x}\right) \\ z \end{pmatrix} \quad (11)$$